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MINKOWSKI SET OPERATORS

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Paper brings few ideas about a concept of Minkowski combinations of point sets, which can be defined as generalisation of Minkowski set operations of point sets in the Euclidean space E^n. Minkowski sum and product of two point sets are used for definition of Minkowski set operators as special mappings in E^n, which can be used for modelling of manifolds in this space.

Concept of Minkowski linear combination of two point sets is an analogy of the well defined concept of a linear combination of two vectors. It can be defined on the basis of the scalar multiple of a point set, which is a point set $A_k \subset \boldsymbol{E}^n$ with elements in all such points of the space, whose positioning vectors are k-multiples of positioning vectors of all points of the set A

$$A_k = k.A = \{k.m : m \in A, m \mapsto \mathbf{m} \Rightarrow k.m \mapsto k.\mathbf{m}\}, k \in R \tag{1}$$

Minkowski sum linear combination of two point sets A and B in the space $\boldsymbol{E}^n$ is point set C in the space $\boldsymbol{E}^n$ defined as follows

$$C = k.A \oplus l.B = A_k \oplus B_l = \{k.a + l.b, a \in A, b \in B\}, k, l \in R \tag{2}$$

Mapping $L_{k,l}$ in which any ordered pair of point sets is related to a linear combination of the two sets (2) will be called Minkowski linear set operator

$$L_{k,l} : (A, B) \rightarrow C = A_k \oplus B_l, k, l \in R \tag{3}$$

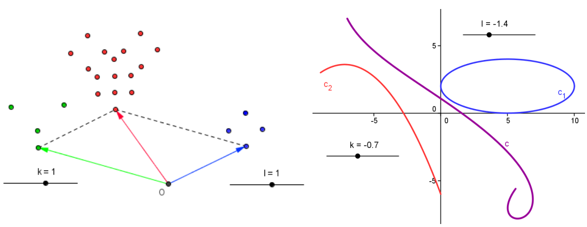

Fig. 1 Minkowski linear combination of discrete point sets and of 2 planar curves

Minkowski product combination of two point sets A and B in $\boldsymbol{E}^n$ is point set C in $\boldsymbol{E}^n$

$$C = k.A \otimes l.B = A_k \otimes B_l = \{k.a \wedge l.b, a \in A, b \in B\}, k, l \in R \tag{4}$$

which is the $k.l$ multiple of the Minkowski product $A \otimes B$ of point sets A and B.

Mapping $LP_{k,l}$ in which any ordered pair of point sets A and B is related to a product combination of the two sets (4) will be called Minkowski product set operator

$$LP_{k,l} : (A, B) \rightarrow C = A_k \otimes B_l, k, l \in R \tag{5}$$

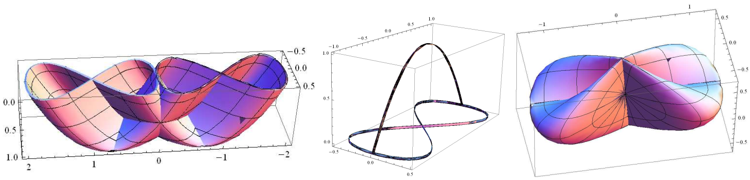

Fig. 2 Minkowski linear and product combination of 2 space curves

Concept of Minkowski arithmetic combination of three sets can be introduced based on Minkowski sum and product. For any point sets A, B, C in $\boldsymbol{E}^n$ and real numbers k, l, h we define Minkowski mixed combination of point sets A, B, C as the following point set W in $\boldsymbol{E}^n$

$$W = (k.A \oplus l.B) \otimes h.C = (A_k \oplus B_l) \otimes C_h = \\ = \{(k.a + l.b) \wedge h.c, a \in A, b \in B, c \in C\} \tag{6}$$

Minkowski arithmetic combination $(k.A \oplus l.B) \otimes h.C$ of three point sets A, B and C in $\boldsymbol{E}^n$ can be represented as the h-multiple of the Minkowski sum of the Minkowski product combinations of sets A and C, and B and C

$$(A_k \oplus B_l) \otimes C_h = ((A \otimes C)_k \oplus (B \otimes C)_l)_h, h, k, l \in R \tag{6}$$

Mapping $LA_{k,l,h}$ in which any ordered triple of point sets A, B and C is related to an arithmetic combination of the three sets (6) will be called Minkowski arithmetic set operator

$$LA_{k,l,h} : (A, B, C) \rightarrow W = (A_k \oplus B_l) \otimes C_h, k, l, h \in R \tag{7}$$

References:

[1] Velichová D: Minkowski Sum in Geometric Modelling, Proc. of the 6th Conference "Geometry and Graphics", Ustroň, Poland, p. 65-66, 2009.

[2] Velichová D: Minkowski Set Operations in Modelling of Manifolds, *Proc. of the GeoGra 2012 Int. Conf.*, Budapest, Hungary, ISBN 978-963-08-3162-8, CD-rom, 4pp., 2012.