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GENERALIZED SLIT SPACES

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Two definitions of an affine space can be found in the literature: it is a projective space with one of its hyperplanes removed, or it is a projective space with one of its hyperplanes distinguished. It is also pretty well known that these two definitions are equivalent, since the removed hyperplane can be recovered in terms of so obtained affine space, and the whole projective space can be completed. So, a general question arises: how big the remaining fragment of a projective space must be so as the surrounding space can be recovered in terms of the internal geometry of this fragment?

There are more examples where removing a hyperplane is also successful and where in result we get some affine geometries. In a polar space deleting its geometric hyperplane yields an affine polar space (cf. [1], [4]). When we delete any subspace, not necessarily a hyperplane, in a projective space, then so called slit space arises, possessing both projective and affine properties (cf. [2], [3]). We deal with a case more general than a slit space: a projective space M with removed any point subset W - a horizon. Lines of M, that are entirely contained in W, are also removed. In the remaining fragment of M, that we call a complement, we introduce a parallelism: two lines are parallel if they meet in the horizon W. Two disjoint classes of maximal cliques of parallelism can be distinguished. Both of them are definable in terms of the complement. One of them, that we call a star direction, consists of lines passing through the fixed point in W. We recover removed points by identifying them with star directions. Next we investigate planes of our complement. It turns out, that there is a triangle with sides, which are not totally removed, on every such plane. Thus, all considered planes are definable in the complement. We use planes sections to recover missing lines. All our research are done under the following assumption:

(⁎) on every line in M the number of removed points is less or equal to the number of remaining points minus two.

Finally we prove

Theorem: If M is a projective space of a dimension at least 3, W is a point subset of M, and M satisfies (⁎), then both M and W can be recovered in the complement of W in M.

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