

## ENTASIS – SHAPE OF BEAUTY

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**Abstract.** Two algorithms for the construction of entasis given by [2,3] are examined in this study. It has been shown, that those shapes are not an ellipse for which they were considered by the authors, but another curves.

**Keywords:** entasis, history of architecture, conic section, Pascal's theorem, collineation

### 1 Entasis in theory of architecture

Interest in the *heritage* of the ancient Greeks, which had been aroused in Italy during the Renaissance, brought the first descriptions of architectural ruins, monuments and ancient sculptures. Since rediscovering of the Vitruvius' *De architectura libri decem* in 1415 a number of books and treatises have been written on the subject of classical architecture.

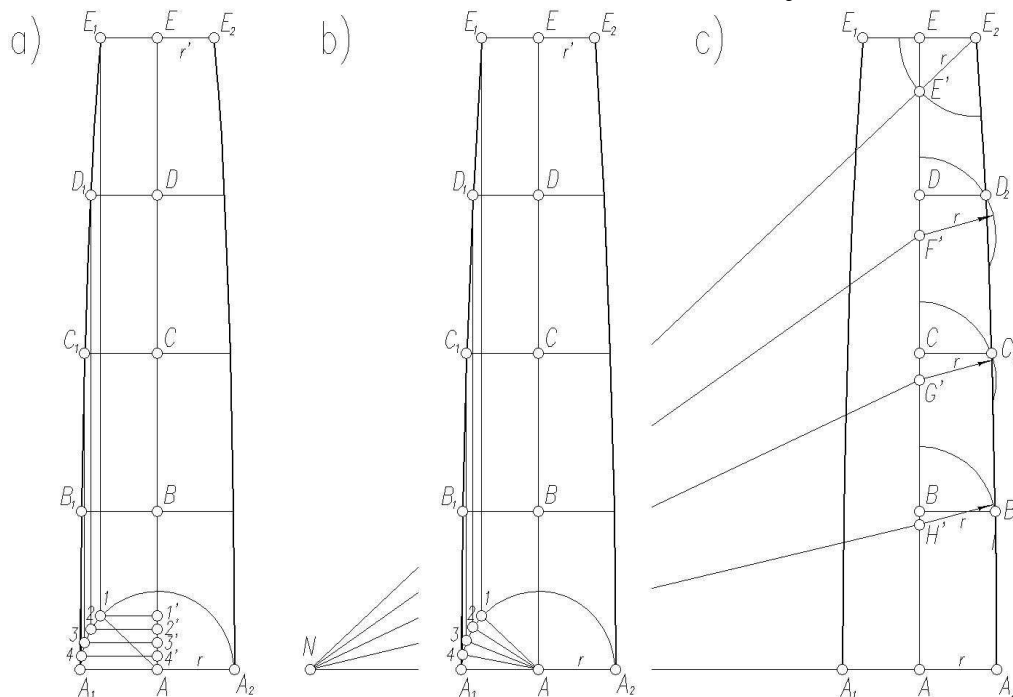


Figure 1: Entasis shaped with different algorithms: a) method given by Thomä [1], b) method I and c) method II both given by Bühlmann and Ulatowski [2, 3]

Systematical, detailed studies of ancient monuments were initiated by founded in 1663 French scientific society – Academie des Inscriptions et Belles-Lettres. Continuation of this research through the years and centuries brought the detailed arrangements for the size, proportion

of individual elements of architecture and geometric construction proposals reflecting their shape. Walter Thomä mentions in his work [1] about 33 books concerning this subject written between 1452 and 1771. Among them 10 focuses exclusively on shape of columns in classical orders.

One of the most popular academic textbooks of the last century is the work of the Technical University of Munich professor Josef Bühlmann – *Die Bauformenlehre* [1]. There are two algorithms for the construction of Doric columns entasis given there (quoted in Polish literature by Ulatowski [3]), which will be examined in this study.

## 2 Entasis of Doric column according to Bühlmann and Ulatowski [2, 3]

Doric column doesn't have a base nor plinth and is composed of only two parts: the stem and the capital. The stem tapers upwards quite significantly, the lower section of the column diameter equal to two modules (which is 60 parties), the top is generally 44 parties, therefore 44:60. The bulge (entasis) moderate. The methods for defining entasis given by authors are presented below.

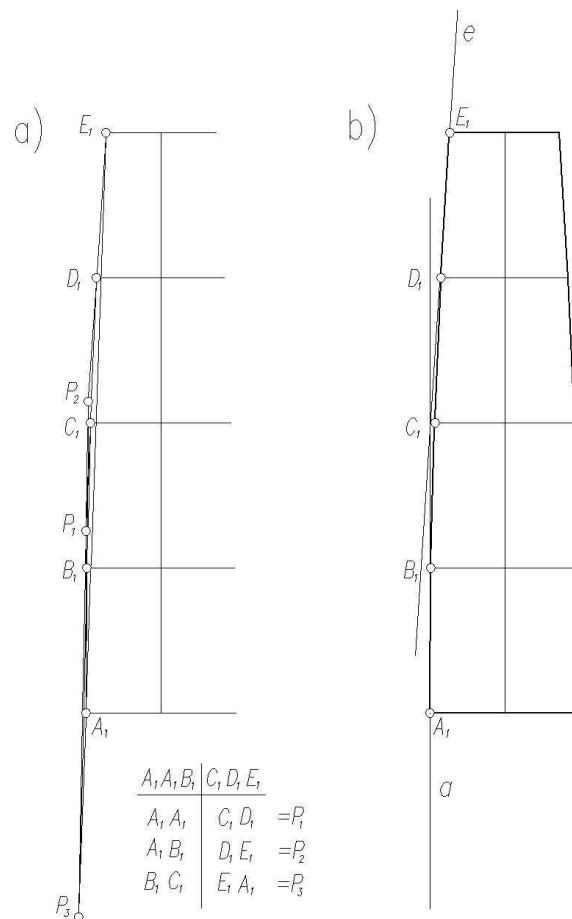


Figure 2: Construction of tangents

Method I (Fig. 1b). One should draw a bottom diameter  $A_1-A_2$ , the central division of the shank and at the top the upper diameter  $E_1-E_2$ . Vertical line that is the height of the column is divided into several equal parts, in this case four ( $A, B, C, D, E$ ). A semicircle should be drawn at the bottom diameter, and from the end of upper diameter ( $E_1$ ) a vertical line to the

circumference of semicircle (1). Thus obtained segment of semicircle ( $A_1 - I$ ) be divided into as many equal angles as the number of equal particles in which the shank height was divided, in that case – four. The dividing points on the section semicircle have numbers 1, 2, 3, 4. From the points 2, 3 and 4 one should draw vertical lines up to the intersection with the corresponding horizontal line from points B, C, D. Note that the distance between the vertical lines are not equal. A line connecting all five points:  $A_1, B_1, C_1, D_1, E_1$ , forms entasis of the stem. It can be seen that this is essentially a *construction of sinusoid* “stretched” by a some factor.

Method II (Fig. 1c). There is the bottom diameter  $A_1 - A_2$ , the central line of the shank  $A - E$  and at the top the upper diameter  $E_1 - E_2$ . Central line is divided into any number of equal parts, in this case four ( $A, B, C, D, E$ ). A circle should be drawn with a center in point  $E_2$ , end of the upper diameter, and radius  $r$  (equal to half of the lower diameter), the circle intersects vertical line at point  $E'$ . Extend the line  $E_2 - E'$  to the point  $N$ , which is the point of intersection with the extension of the lower diameter. Section of vertical line  $A - E'$  should be divided into the same number of equal parts into which the line segment  $A - E$  was divided ( $F', G', H'$ ). From the point of intersection  $N$  one should pull rays to the points of intersection  $F', G', H'$ . In these points one should draw circles with radius  $r$ , intersections of the circles with horizontal lines from points B, C, D gives points  $B_2, C_2, D_2$ . Connection of points  $A_2, B_2, C_2, D_2, E_2$ , forms entasis.

It can be seen the construction is very similar to the construction of conchoids of Nikomedes –  $(x^2 + y^2)(x - a)^2 = d^2 x^2$ , where  $a = NA$ , and  $d = r$ . Interestingly, the authors [2,3] giving the algorithms of both methods didn't mention their mathematical interpretation, which more, states that obtained in this way curves are fragments of an ellipse.

### 3 Entasis' geometry analysis

Both constructions presented in [2,3] are based on the separation of column height into any number of equal parts. By setting the division into 4 parts, there would be 5 points obtained designating the shape of the column. Five points lying on a plane determines a conic curve. So the curve defining entasis of Doric columns, determined in this way, *of course, can be seen as a part of conic section*.

To determine type of the conic section one should look for its characteristic parameters. Using the Pascal's theorem about a hexagon inscribed in a conic section [4] additional tangents (named as a and e) at points ( $A_1$  &  $E_1$ ) have been constructed (Fig. 2). The following cyclic order was set for conical points:  $A_1, B_1, C_1, D_1$  and  $E_1$ . In this case, the opposite sides of a hexagon inscribed in a conic ( $A_1 A_1$  &  $C_1 D_1, A_1 B_1$  &  $D_1 E_1, B_1 C_1$  &  $E_1 A_1$ ) intersect at points  $P_1, P_2, P_3$  (lying on the Pascal's line). When setting the points  $P_2$  and  $P_3$  we get the Pascal's line, and the point  $P_1$  on the line as the intersection with the side  $C_1 D_1$ . The tangent at the point  $A_1$  passes through  $P_1$  (as the hexagon's side  $A_1 A_1$ ). In the same way tangent e can be determined.

Basing on the two points, tangents at these points ( $A_1, a, E_1, e$ ) and the third point ( $C_1$ ) one can determine the collineation project the curve (conic section) into the circle  $k$  (collineation axis  $t$  determined at the tangent  $a$ , circle of freely chosen size tangentially to  $a$  in the point  $A_1$ ).

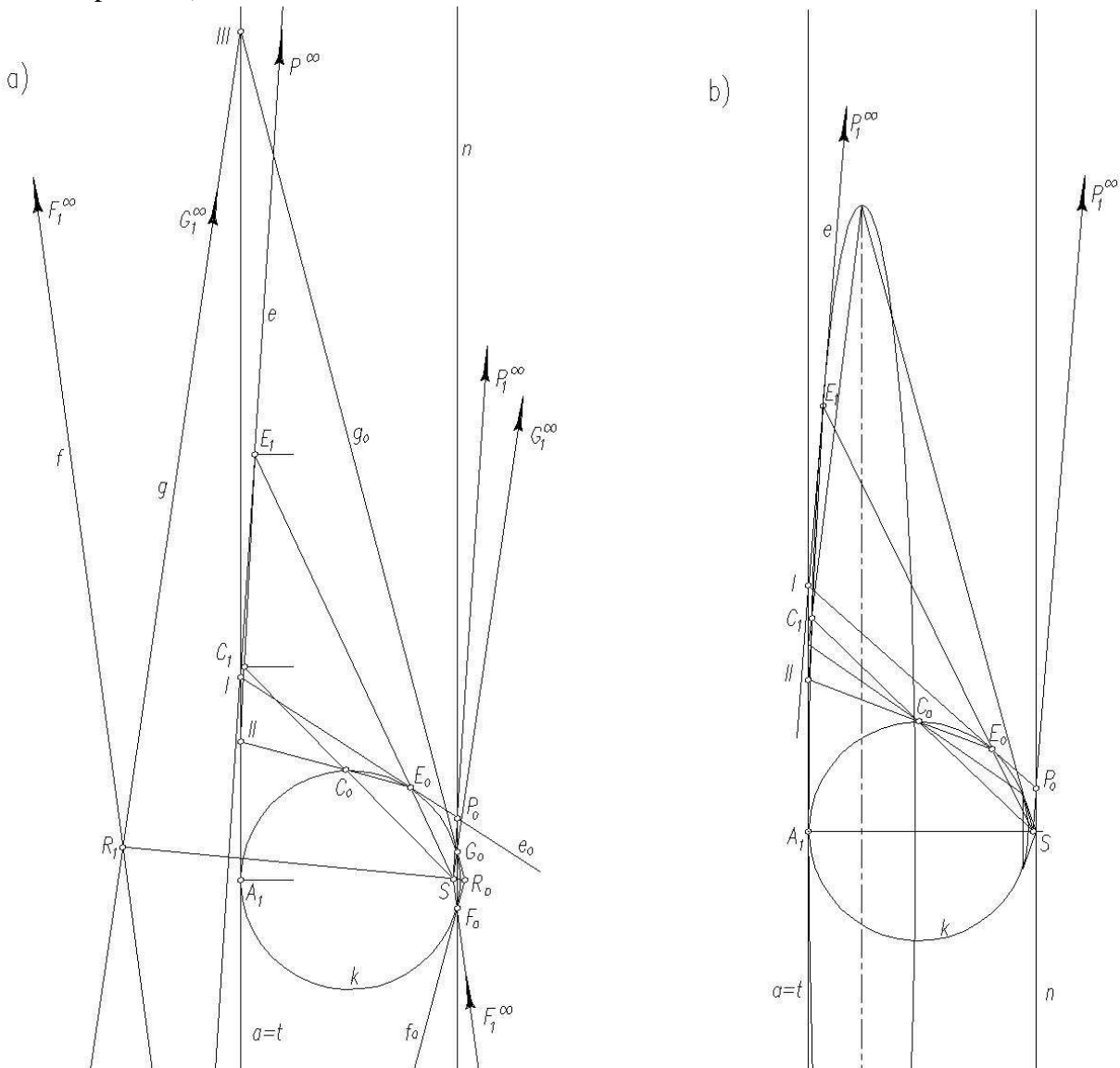


Figure 3: Demonstration of entasis shape as a segment of: a) hyperbola and b) ellipse

To construct a tangent to the circle ( $e_0$ ) from the point I on the collineation axis, one sets point  $E_0$ , and leads line  $E_1C_1$  and finds its equivalent  $E_0C_0$  in the circle's system. Couples of points  $E_1, E_0; C_1, C_0$  define the center of collineation  $S$ . Picking point at infinity  $P_1$  on the line  $e$ , one can determine a border line passing through  $P_0$ . That allows one to state clearly the type of the curve. Because when the entasis is shaped by method I or II, the border line  $n$  crosses the circle  $k$  it will be a hyperbola, not ellipse as given by Bühlmann [2] and Ulatowski [3] (Fig. 3a).

Whereas the shape of the ellipse (straight boundary does not intersect the circle – Fig. 3b), would be obtained in the case of application method given by Walter Thomä [1] (equidistance between the points of  $1', 2', 3', 4'$ , Fig. 1a), assuming of course, that the analysis we submit 5 points of construction.

It is interesting that by far the non-compliances weren't noticed nor corrected. Given the high level of development of projective geometry in Germany in the nineteenth century and attention paid to details by German authors one can be surprised. We hope that popularization of this issue among specialists both the geometry and the history of architecture will broaden our knowledge on this topic.

### References

- [1] Thomä W.: *Die Schwellung der Säule (Entasis) bei den Architekturtheoretikern bis in das XVIII. Jahrhundert*, Dresden, 1915.
- [2] Bühlmann J.: *Die Bauformenlehre*. Verlag von Arnold Bergsträsser, Darmstadt, 1896, p. 65.
- [3] Ulatowski K.: *Architektura starożytnej Grecji*. PWN, Warszawa-Poznań, 1962, 73–74.
- [4] Otto F., Otto E.: *Podręcznik geometrii wykreslonej*. PWN, Warszawa, 1975, 194–208.

## ENTAZIS – KSZTAŁT PIĘKNA

Zainteresowania spuścizną starożytnych Greków, które zostały rozbudzone w okresie Renesansu, zainspirowały całe pokolenia architektów i teoretyków architektury do prowadzenia szczegółowych badań. Zaowocowało to ustaleniami dotyczącymi wielkości i proporcji poszczególnych elementów architektury, a także propozycjami konstrukcji geometrycznych odwzorowujących ich kształt. Jedną z popularniejszych publikacji dotyczących tych zagadnień jest praca [2], w której podano dwie konstrukcje kształtu entasis kolumny doryckiej. Jak można zauważyć, jest to konstrukcja sinusoidy („rozciągniętej” o pewien współczynnik) oraz konstrukcja konchoidy Nikomedesa. Żaden z autorów nie analizuje jednak matematycznej strony zagadnienia, co więcej podają oni, że kształt entasis to fragment elipsy. Nawet przy założeniu, że do analizy weźmiemy 5 punktów i posłużymy się kolineacją, to okazuje się, że przechodząca przez nie stożkowa jest bardziej zbliżona do hiperboli, niż do elipsy. Interesujące jest, że tych niezgodności jak dotąd nikt nie zauważył i nie sprostował.